Answer Set Programming: A Declarative Approach to Solving Challenging Search Problems

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Aalto University (http://www.aalto.fi/en/) established in 2010

Merger of three leading Finnish universities in their fields:

- Helsinki School of Economics
- Helsinki University of Technology
- University of Art and Design Helsinki.

Unique combination of science and art with business and technology.
Answer Set Programming (ASP)

- Basic principles outlined in the late 1990s
- Now well represented at research conferences and workshops (IJCAI, AAAI, ECAI, KR, ...)
- Competitive implementations available (winning first places even in SAT competitions 2009, 2011)
- Growing number of applications
- An approach to modeling and solving knowledge intensive search problems with defaults, exceptions, definitions:
  planning, configuration, model checking, network management, linguistics, bioinformatics, combinatorics, ...
Content

- Introduction to Answer Set Programming (ASP)
- Stable Model Semantics
- Solving Problems with ASP
- ASP Solver Technology
- Systems, Applications, Literature
Part I

Introduction to ASP
Answer Set Programming

- Term coined by Vladimir Lifschitz in the late 1990s.
- Roots: KR, logic programming, nonmonotonic reasoning.
- Based on some formal system with semantics that assigns a theory (program/set of constraints) a collection of answer sets (models).
- An ASP solver: computes answer sets for a theory.
- Solving a problem in ASP:
  Encode the problem as a theory such that solutions to the problem are given by answer sets of the theory.
ASP—cont’d

▶ Solving a problem using ASP

Problem → instance Encoding → Theory → ASP solver → Models → (Solutions)

▶ Possible formal system Models

<table>
<thead>
<tr>
<th>Formal System</th>
<th>Models</th>
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<td>Propositional logic</td>
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<td>CSP</td>
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▶ Similar to constraint programming, MIP, SAT applications (SAT planning, symbolic model checking), . . .
**Example.** *k*-coloring problem with SAT

- Given a graph \((V, E)\) find an assignment of one of \(k\) colors to each vertex such that no two adjacent vertices share a color.

- **Encoding 3-coloring using propositional logic**
  - For each vertex \(v \in V\) include the clauses:
    \[
    v_1 \lor v_2 \lor v_3 \\
    \neg v_1 \lor \neg v_2 \\
    \neg v_1 \lor \neg v_3 \\
    \neg v_2 \lor \neg v_3
    \]
  - and for each edge \((v, u) \in E\) the clauses:
    \[
    \neg v_1 \lor \neg u_1 \\
    \neg v_2 \lor \neg u_2 \\
    \neg v_3 \lor \neg u_3
    \]

- 3-colorings of a graph \((V, E)\) and models of the encoding correspond: vertex \(v\) colored with color \(i\) iff \(v_i\) true in a model.
ASP Using Logic Programs

- Uniform encoding: separate problem specification and data
- Compact, easily maintainable representation
- Integrating KR, DB, and search techniques
- Handling dynamic, knowledge intensive applications: data, frame axioms, exceptions, defaults, closures, inductive definitions
Coloring Problem (Uniform Encoding)

% Problem encoding
1 { colored(V,C):color(C) } 1 :- vtx(V).
:- edge(V,U), color(C), colored(V,C), colored(U,C).

% Data
vtx(a). ...
edge(a,b). ...
color(r). color(g). ...

Legal colorings of the graph given as data and stable models of the problem encoding and data correspond:
a vertex \( v \) colored with a color \( c \) iff \( \text{colored}(v, c) \) holds in a stable model.
What is ASP Good for?

**Knowledge intensive search problems** with defaults, exceptions, inductive definitions:

- Constraint satisfaction
- Planning, routing
- Computer-aided verification
- Security analysis
- Linguistics
- Network management
- Product configuration
- Combinatorics
- Diagnosis
ASP Using Logic Programs

- Logic programming: framework for merging KR, DB, and search
- PROLOG style logic programming systems not directly suitable for ASP:
  - search for proofs (not models) and produce answer substitutions
  - not entirely declarative
- In late 80s new semantical basis for “negation-as-failure” in LPs based on nonmonotonic logics: Stable model semantics
- Implementations of stable model semantics led to ASP
  - Smodels [N. and Simons 1996]
  - Basic ASP principles [N. 1999; Marek and Truszczyński 1999]
  - The term ASP coined by V. Lifschitz in 1999
Part II

Stable Model Semantics
LPs with Stable Models Semantics

Consider first normal logic program rules

\[ A \leftarrow B_1, \ldots, B_m, \text{not } C_1, \ldots, \text{not } C_n \]

Seen as constraints on an answer set (stable model):

- if \( B_1, \ldots, B_m \) are in the set and
- none of \( C_1, \ldots, C_n \) is included,
then \( A \) must be included in the set

A stable model is a set of atoms

(i) which satisfies the rules and
(ii) where each atom is **justified** by the rules

(negation by default; CWA)
Stable Models — cont’d

- Program:
  \[ b \leftarrow \]
  \[ f \leftarrow b, \text{not } eb \]
  \[ eb \leftarrow p \]

- Stable model: \( \{ b, f \} \)

- Another candidate model: \( \{ b, eb \} \)
  satisfies the rules but is not a proper stable model:
  \( eb \) is included for no reason.

- Justifiability of stable models is captured by the notion of a **reduct** of a program.

  The stable model semantics [Gelfond/Lifschitz, 1988].
Definite Programs

- For the reduct we need to consider first definite programs, i.e. normal programs without negation (not ).
- Such a program $P$ has a unique least model $LM(P)$ satisfying the rules.
- $LM(P)$ can be constructed, e.g., by forward chaining.

Examples.

$P_1 :$

\[
\begin{align*}
  p & \leftarrow \\
  q & \leftarrow p
\end{align*}
\]

$LM(P_1) = \{p, q\}$

$P_2 :$

\[
\begin{align*}
  p & \leftarrow q \\
  q & \leftarrow p
\end{align*}
\]

$LM(P_2) = \{\}$

$P_3 :$

\[
\begin{align*}
  p & \leftarrow q \\
  q & \leftarrow p \\
  p & \leftarrow
\end{align*}
\]

$LM(P_2) = \{p, q\}$
Consider the propositional (variable free) case:
- $P$ — ground program
- $S$ — set of ground atoms

Reduct $P^S$ (Gelfond-Lifschitz)
- delete each rule having a body literal not $C$ with $C \in S$
- remove all negative body literals from the remaining rules

$P^S$ is a definite program (and has a unique least model $LM(P^S)$)

$S$ is a stable model of $P$ iff $S = LM(P^S)$. 
Example. Stable models

<table>
<thead>
<tr>
<th>$S$</th>
<th>$P$</th>
<th>$P^S$</th>
<th>$\text{LM}(P^S)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>${b, f}$</td>
<td>$b \leftarrow$</td>
<td>$b \leftarrow$</td>
<td>${b, f}$</td>
</tr>
<tr>
<td></td>
<td>$f \leftarrow b, \text{not} eb$</td>
<td>$f \leftarrow b$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$eb \leftarrow p$</td>
<td>$eb \leftarrow p$</td>
<td></td>
</tr>
<tr>
<td>${b, eb}$</td>
<td>$b \leftarrow$</td>
<td>$b \leftarrow$</td>
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<tr>
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<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>$eb \leftarrow p$</td>
<td>$eb \leftarrow p$</td>
<td></td>
</tr>
</tbody>
</table>

- The set $\{b, eb\}$ is not a stable model of $P$ but $\{b, f\}$ is the (unique) stable model of $P$
Example. Stable models

- A program can have none, one, or multiple stable models.

- Program:
  
  $p \leftarrow \text{not } q$
  
  $q \leftarrow \text{not } p$

  Two stable models:
  
  $\{p\}$
  
  $\{q\}$

- Program:
  
  $p \leftarrow \text{not } p$

  No stable models
Programs with variables

- Variables are needed for uniform encodings
- Semantics: **Herbrand models**
- A rule is seen as a shorthand for the set of its ground instantiations over the Herbrand universe of the program
- The **Herbrand universe** is the set of terms built from the constants and functions in the program

Database assumptions: unique names, domain closure
Example. Programs with variables

- For the program $P$:
  
  edge(1,2).
  edge(1,3).
  edge(2,4).
  path(X,Y) :- edge(X,Y).
  path(X,Y) :- edge(X,Z), path(Z,Y).

  The Herbrand universe is \{1,2,3,4\}.

- Hence, the rule path(X,Y) :- edge(X,Y). in $P$ represents the set of ground instantiations:
  
  path(1,1) :- edge(1,1).
  path(1,2) :- edge(1,2).
  path(2,1) :- edge(2,1).
  path(2,2) :- edge(2,2).
  path(1,3) :- edge(1,3).

  ...
Stable Models — cont’d

- A stratified program (no recursion through negation) has a unique stable model (canonical model).
- It is **linear time to check** whether a set of atoms is a stable model of a ground program.
- It is **NP-complete to decide** whether a ground program has a stable model.
- Normal programs (without function symbols) give a **uniform encoding** to every NP search problem.
Extensions to Normal Programs

- An integrity constraint is a rule without a head:
  
  \[ \leftarrow B_1, \ldots, B_m, \text{not } C_1, \ldots, \text{not } C_n \]

- It can be seen as a shorthand for
  
  \[ F \leftarrow \text{not } F, B_1, \ldots, B_m, \text{not } C_1, \ldots, \text{not } C_n \]

- and it eliminates stable models where the body
  \[ B_1, \ldots, B_m, \text{not } C_1, \ldots, \text{not } C_n \]
  is satisfied.

- Classical negation can be handled by normal programs (renaming):
  
  \[ p \leftarrow \text{not } \neg p \quad \text{corresponds to} \quad p \leftarrow \text{not } p' \]
  
  \[ \leftarrow p, p' \]
Extensions to Normal Programs

- **Encoding of choices**
  - A key point in ASP
  - Choices can be encoded using normal rules with unstratified negation
    
    \[
    a \leftarrow \neg a', b, \neg c
    \]
    
    \[
    a' \leftarrow \neg a
    \]

- **Choice rules**, however, provide a much more intuitive encoding:
  
  \[
  \{a\} \leftarrow b, \neg c
  \]

- **Disjunctive rules**: \( a \lor a' \leftarrow b, \neg c \)
  - Higher expressivity and complexity (\( \Sigma^p_2 \))
  - Special purpose implementations (dlv, claspD)
  - Can be implemented also using an ASP solver for normal programs as the **core engine** (GnT)
Extensions — cont’d

- Many extensions implemented using an ASP solver as the core engine:
  - preferences
  - nested logic programs
  - circumscription, planning, diagnosis, ...
  - HEX-programs
  - DL-programs

- Aggregates (count, sum, …)
- Optimization
- Function symbols
- Built-in predicates and functions:

  ```prolog
  nextstate(Y,X) :- time(X), time(Y), Y = X + 1.
  ```
Example. Rules in lparse

- **Cardinality constraints**
  
  \[ 2 \{ \text{hd}_1, \ldots, \text{hd}_n \} 4 \]

- **Weight constraints**
  
  \[ 200 [ \text{hd}_1 = 60, \ldots, \text{hd}_n = 130 ] \]

  A.k.a. **pseudo-Boolean constraints**:

  \[ 60\text{hd}_1 + \cdots + 130\text{hd}_n \geq 200 \]

- **Optimization**

  minimize \[ [ \text{hd}_1 = 100, \ldots, \text{hd}_n = 180 ] \].

- **Conditional literals:**

  expressing sets in cardinality and weight constraints

  \[ 1 \{ \text{colored}(V,C) : \text{color}(C) \} 1 :- \text{vtx}(V). \]
Part III

Solving Problems using ASP
Programming Methodology

- Uniform encodings: separate data and problem encoding
- Basic methodology: \textbf{generate and test}
  - \textbf{Generator rules}: provide candidate answer sets (typically encoded using choice constructs)
  - \textbf{Tester rules}: eliminate non-valid candidates (typically encoded using integrity constraints)
  - \textbf{Optimization statements}: Criteria for preferred answer sets (typically using cost functions)
Example: Coloring

% Problem encoding

% Generator rule
1 {colored(V,C):color(C)} 1 :- vtx(V).

% Tester rule
:- edge(V,U), color(C), colored(V,C), colored(U,C).

% Optimization statement
minimize {colored(V,y):vtx(V)}.

% Data
vtx(a). ...
edge(a,b). ...
color(r). color(g). color(b). color(y).
Example: Review assignment

% Data
reviewer(r1),...
paper(p1), ...
classA(r1,p1), ... % Preferred papers
classB(r1,p2), ... % Doable papers
coi(r1,p3), ... % Conflicts of interest

% Problem encoding

% Generator rule
% Each paper is assigned 3 reviewers
3 { assigned(P,R):reviewer(R) } 3 :- paper(P).
% Tester rules
% No paper assigned to a reviewer with coi
:- assigned(P,R), coi(R,P).
% No reviewer has an unwanted paper.
:- paper(P), reviewer(R),
    assigned(P,R), not classA(R,P), not classB(R,P).
% No reviewer has more than 8 papers
:- 9 \{ assigned(P,R): paper(P) \}, reviewer(R).
% Each reviewer has at least 7 papers
:- \{ assigned(P,R): paper(P) \} 6, reviewer(R).
% No reviewer has more than 2 classB papers
:- 3 \{ assignedB(P1,R): paper(P1) \}, reviewer(R).
assignedB(P,R) :- classB(R,P), assigned(P,R).
% Minimize the number of classB papers
minimize [ assignedB(P,R):paper(P):reviewer(R) ].
Fixed Points

- The stable model semantics captures inherently **minimal fixed points** enabling compact encodings of **closures and inductive definitions**

- **Example.** Reachability from node $s$.

  \[
  r(s).
  \]

  \[
  r(V) : \text{edge}(U,V), r(U).
  \]

  \[
  \text{edge}(a,b). \ldots
  \]

- The program captures reachability: it has a unique stable model $S$ s.t. $v$ is reachable from $s$ iff $r(v) \in S$.

- **Example.** Transitive closure of a relation $q(X, Y)$

  \[
  t(X,Y) : - q(X,Y).
  \]

  \[
  t(X,Y) : - q(X,Z), t(Z,Y).
  \]
ASP vs Other Approaches

▶ SAT, CSP, (M)IP
  ▶ Similarities: search for models (assignments to variables) satisfying a set of constraints.
  ▶ Differences: no logical variables, fixed points, database, DDB or KR techniques available, search space given by variable domains.

▶ LP, CLP:
  ▶ Similarities: database and DDB techniques.
  ▶ Differences: Search for proofs (not models), non-declarative features.
Part IV

ASP Solver Technology
ASP Solvers

- ASP solvers need to handle two challenging tasks
  - complex data
  - search
- The approach has been to use
  - logic programming and deductive data base techniques for the former
  - SAT/CSP related search techniques for the latter
- In the current systems: separation of concerns
  - A two level architecture
Architecture of ASP Solvers

Typically a two level architecture employed

- **Grounding** step handles complex data:
  - Given program $P$ with variables, generate a set of ground instances of the rules which preserves the models.
  - LP and DDB techniques employed.

- **Model search** for ground programs:
  - Special-purpose search procedures
  - Exploiting SAT/SMT solver technology
Typical ASP System Tool Chain

program (variables) → Grounder → ground program → Model finder → stable models

- **Grounder:**
  - (deductive) DB techniques
  - built-in predicates/functions (e.g. arithmetic)
  - function symbols

- **Model finder:**
  - SAT technology (propagation, conflict driven clause learning)
  - Special propagation rules for rules
  - Support for cardinality and weight constraints and optimization built-in
SAT and ASP

ASP systems have much more expressive modelling languages than SAT: variables, built-ins, aggregates, optimization

For model finding for ground normal programs results carry over: efficient unit propagation techniques, conflict driven learning, backjumping, restarting, . . .

ASP model finders have special (unfounded set based) propagation rules for recursive rules

ASP model finders have built-in support for aggregates (cardinality and weight constraints) and optimization

One pass compact translations to SAT and SMT available: progress in SAT and SMT solver technology can also be exploited directly in ASP model finding.
Part V

Systems, Applications, Literature
Some ASP Systems

**Grounders:**
dlv [http://www.dbai.tuwien.ac.at/proj/dlv/](http://www.dbai.tuwien.ac.at/proj/dlv/)
XASP with XSB [http://xsb.sourceforge.net](http://xsb.sourceforge.net)

**Model finders (disjunctive programs):**
dlv [http://www.dbai.tuwien.ac.at/proj/dlv/](http://www.dbai.tuwien.ac.at/proj/dlv/)
Some ASP Systems

Model finders (non-disjunctive programs):
ASSAT  http://assat.cs.ust.hk/
clasp  http://potassco.sourceforge.net/
CMODELS  http://userweb.cs.utexas.edu/users/tag/cmodels/
LP2DIFF  http://www.tcs.hut.fi/Software/lp2diff/
LP2SAT  http://www.tcs.hut.fi/Software/lp2sat/
Smodels  http://www.tcs.hut.fi/Software/smodels/
SUP  http://userweb.cs.utexas.edu/users/tag/sup/

▶ For systems, performance, benchmarks, and examples, see for instance the latest ASP competition:
http://dtai.cs.kuleuven.be/events/ASP-competition/
Applications

- Planning
  For example, USAdvisor project at Texas Tech:
  A decision support system for the flight controllers of space shuttles

- Product configuration
  – Intelligent software configurator for Debian/Linux
  – WeCoTin project (Web Configuration Technology)
  – Spin-off (http://www.variantum.com/)

- Computer-aided verification
  – Partial order methods
  – Bounded model checking
Applications—cont’d

- Data and information Integration
- Semantic web reasoning
- Team building at Gioia Tauro Seaport
- Repairing large-scale biological networks
- ASP-based music composition system (anton-demo.wav)
- VLSI routing, planning, combinatorial problems, network management, network security, security protocol analysis, linguistics …
- WASP Showcase Collection
  http://www.kr.tuwien.ac.at/research/projects/WASP/showcase.html
Some Literature


Conclusions

ASP = KR + DB + search

- ASP emerging as a viable KR tool
- Efficient implementations under development
- Expanding functionality and ease of use
- Growing range of applications
Topics for Further Research

- Intelligent grounding
- Model computation without full grounding
- Program transformations, optimizations
- Model search
- Distributed and parallel implementation techniques
- Language extensions
- Programming methodology
- Testing techniques
- Tool support: debuggers, IDEs